RENAISSANCE MECHANICS AND THE NEW SCIENCE OF MOTION*

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In his pioneering work *Les origines de la statique* (1905-06), Pierre Duhem not only inaugurated the history of medieval science, but he also set in motion the reevaluation of the work of Galileo in the light of his predecessors that continues today. In particular, Duhem recognized in the medieval science of weights (*scientia de ponderibus*) the origin of the idea of virtual velocity, which would prove so fruitful in the rational mechanics of Lagrange, and traced it back to the pseudo-Aristotelian *Mechanical Problems*.[1] Since Duhem, there has come to light a whole tradition in the sixteenth century surrounding the *Mechanical Problems*, which turns out to have been even more influential directly than the medieval tradition of Jordanus. Galileo himself, in his *Discourse on Bodies in Water* of 1612, explicitly credited the pseudo-Aristotelian *Mechanical Problems* with the general principle of the balance, and thus of all mechanics:

> This proportion between weights and speeds is found in all mechanical instruments and was considered by Aristotle as a principle in his *Mechanical Problems*.[2]

* My thanks to Jürgen Renn for his perceptive comments on and criticisms of the earlier version of this paper.

[1] Pierre Duhem, *Les origines de la statique*, 2 vols. (Paris: A. Hermann, 1905-06), tr. Grant F. Leneaux, Victor N. Vagliente, and Guy H. Wagener as *The Origins of Statics*: *The Sources of Physical Theory* (Dordrecht: Kluwer, 1991).

[2] Galileo Galilei, *Discourse on Bodies in Water*, in *Opere di Galileo Galilei,* ed. Antonio

In the work that followed, he went on to apply this principle to establish the conditions for the equilibrium of submerged bodies. As Mario Helbing in this volume documents, Galileo drew on the *Mechanical Problems* throughout his career, from his earliest works on motion and mechanics written in the 1590s to the culmination of his scientific career in the *Discourses on Two New Sciences* of 1638.

Nevertheless, Alexandre Koyré, writing 35 years after Duhem, denied Duhem's medieval influences and argued persuasively that Galileo had explicitly abandoned the traditional causal and qualitative approach, characteristic of Aristotelian and medieval natural philosophy and mechanics, to found an entirely new science of motion on the model of Archimedean statics. For Koyré, Galileo's new, mathematical physics was an "Archimedean dynamics", the statics of Archimedes set in motion.[3] But how exactly did Galileo set Archimedes in motion? A number of historians of science besides Duhem, including René Dugas, Stillman Drake, and Paul Lawrence Rose, also saw the pseudo-Aristotelian principle as an early form of the principle of virtual velocities, and asserted that it enabled Galileo, in effect, to unify statics and dynamics.[4] From the point of view of the future of virtual velocities in the mechanics of Lagrange and others, this is perhaps *in effect* what Galileo accomplished with this principle, though it is unlikely that he saw it in these terms. Rather, I think he saw himself as working within the tradition of mechanics passed on to him through the sixteenth century, and he saw himself as continuing the attempts of his predecessors to reconcile the various sources of that tradition with each other and to extend them further. In what follows I should like first to sketch the sources of mechanics that converged in the Renaissance and the attempts to unify them, usually under the pseudo-Aristotelian principle. Then I should like to show how Galileo, working within this tradition of renaissance mechancs, applied that principle as causal proof of the single postulate of his new, Archimedean science of accelerated motion. This will suggest, I think, that Galileo's new science of motion was neither so abstractly anti-

Favaro, 20 vols., Edizione Nazionale (Florence: Giunti Barbèra, 1890-1909; rpt. 1964-66), Vol. 4, 69, tr. Thomas Salusbury, in *Mathematical Collections and Translations*, Vol. 2 (London: W. Leybourn, 1665), rpt. in facsimile as Galileo Galileo, *Discourse on Bodies in Water* (Urbana: Univ. Illinois Press, 1960), p. 7; and tr. in Stillman Drake, *Cause, Experiment, and Science* (Chicago: Univ. Chicago Press, 1981), p. 31 (I have slightly altered Drake's translation).

[3] Alexandre Koyré, *Études galiléennes* (Paris: A. Hermann, 1939; 2nd ed., 1969), tr. John Mephan, *Galileo Studies* (New Jersey: Humanities Press), esp. p. 37.

[4] René Dugas, *Histoire de la mécanique* (Paris, 1950), tr. J. R. Maddox, *A History of Mechanics* (New York: Routledge & Kegan Paul, 1957), p. 20; Stillman Drake, *Galileo at Work: His Scientific Biography* (Chicago: Univ. Chicago Press, 1978), pp. 124, 191; see also Drake's introduction to Galileo's *Mecaniche* in I. E. Drabkin and Stillman Drake, *Galileo Galilei on Motion and on Mechanics: Comprising De motu (ca. 1590) and Le Meccaniche (ca. 1600)* (Madison: Univ. Wisconsin Press, 1960), pp 138-139; and Paul Lawrence Rose, *The Italian Renaissance of Mathematics* (Geneva: Droz, 1975), pp. 232-233, 249, 292.

causal nor so anti-Aristotelian as Koyré and others have sometimes made it out to be. It also confirms what Peter Machamer has suggested in a recent article –the great importance for Galileo of the balance, as expressing in its simplest form the general explanatory principle of all mechanical effects.[5] In this Galileo was the culmination of the main stream of sixteenth-century mechanical thought, for all his predecessors, too, saw the balance as embodying the evident physical principle on which a demonstrative science of mechanics must be grounded.

1 The Principle of Circular Movement in Renaissance Mechanics

The mechanical principle to which Galileo appealed had a long history in ancient and renaissance mechanics (see Figure 1). It appeared first in the pseudo-Aristotelian *Mechanical Problems*, which was written in the fourth century BC, probably by a pupil of Aristotle's, though later it was usually attributed to Aristotle himself. This principle, which is called the principle of circular movement or simply Aristotle's mechanical principle, asserts that the farther from the centre of rotation a power or weight is, the faster it will move and the more effective it will be. In the *Mechanical Problems* the principle is applied first to the balance to explain why a lighter weight, farther from the centre and thus having a tendency to move more swiftly on the longer radius, can balance a heavier weight closer to the centre. The balance, in turn, is used to explain other mechanical effects, including the lever, wheel and axel, wedge, hammers, oars, rudders, and the like.[6] The *Mechanical Problems* was apparently unknown through the Latin Middle Ages, though it seems to have had an indirect influence (through Arabic versions and derivatives) on what was called in the medieval Latin West the "science of weights" (*scientia de ponderibus*). In the science of weights, represented mainly by the works of Jordanus de Nemore in the thirteenth century and their later redactions, mechanical effects are similarly explained through an appeal to the speeds of moving powers and weights.[7]

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[5] Peter Machamer, "Galileo's Machines, his Mathematics, and his Experiments", in *The Cambridge Campanion to Galileo*, ed. Peter Machamer (Cambridge: Cambridge Univ. Press, 1998), pp. 53-79.

[6] Pseudo-Aristotle, *Mechanika*, ed. Maria Elisabetta Bottecchia, Studia Aristotelica 10 (Padua: Antenore, 1982); ed. and tr. W. S. Hett, as *Mechanical Problems*, in Aristotle, *Minor Works* (Cambridge, Mass.: Harvard Univ. Press, 1936); the principle is treated at 843b-849b.

[7] Besides Duhem's *Origines de la statique*, see Ernest A. Moody and Marshall Clagett, eds., *The Medieval Science of Weights (Scientia de Ponderibus): Treatises Ascribed to Euclid, Archimedes, Thabit ibn Qurra, Jordanus de Nemore, and Blasius of Parma* (Madison: Univ. Wisconsin Press, 1952), selections from which are also found in Marshall Clagett, *The Science of Mechanics in the Middle Ages* (Madison: Univ. Wisconsin Press, 1959); and Joseph E. Brown, "The *Scientia de Ponderibus* in the Later Middle Ages." Diss. Univ. Wisconsin 1967.

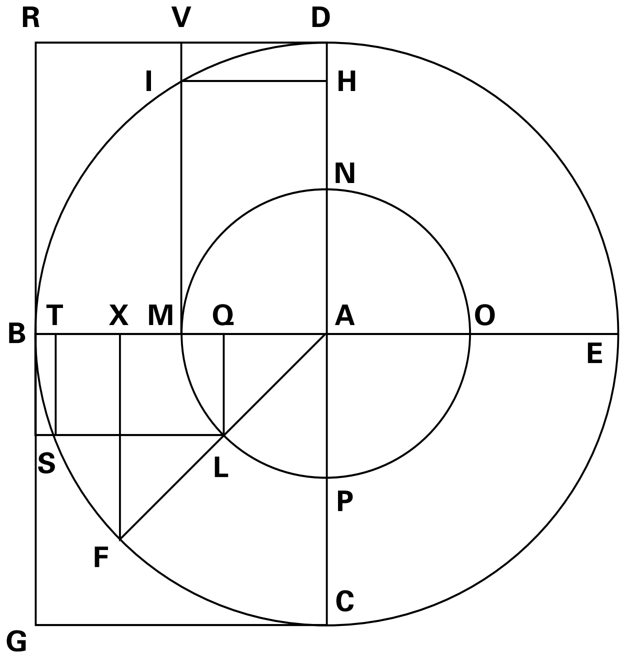

Fig. 1. Aristotle's mechanical principle. Redrawn from Alessandro Piccolomini, In mechanicas quaestiones Aristotelis paraphrasis *(Venice, 1565), f. 15 r.*

In the sixteenth century, these two traditions of theoretical mechanics –the ancient *Mechanical Problems* and the medieval science of weights– were joined by a third, that of Archimedes of Syracuse. In constrast to the other two, the mechanics of Archimedes is purely statical and abstractly mathematical, though, again, the balance is given first place. Archimedes proved the law of equilibrium –that weights on a balance in equilibrium are inversely as their distances from the centre– by resolving unequal weights in equilibrium at unequal distances from the centre into equal weights distributed uniformly in accordance with their centres of gravity, without any appeal to motion or speed at all. Although the works of Archimedes had been translated from Greek to Latin in 1269, they lay apparently almost unread through the late Middle Ages, only to reappear at the end of the fifteenth century, when they suddenly became topical.[8]

But the first mechanical work to attract wide attention in the Renaissance was the pseudo-Aristotelian *Mechanical Problems*, undoubtedly because of the reputation of its alleged author. Cardinal Bessarion had the Greek text copied from a twelfth-century Byzantine manuscript and printed by Aldus

[8] On the medieval and renaissance tradition of Archimedes, see Marshall Clagett, *Archimedes in the Middle Ages*, 5 vols. (Madison: Univ. Wisconsin Press, 1964; Philadelphia: American Philosophical Society, 1976-1984).

Manutius in the first Greek edition of Aristotle in 1495-98. Subsequently the text was translated several times into Latin and commented on by humanists and men of letters, whence it came to the attention of mathematicians and engineers. The story of its recovery and reception has been told elsewhere;[9] here I want only to describe how it figured with the other two traditions in sixteenth-century discussions of the foundations and principles of mechanics. Shortly after the *Mechanical Problems* was widely available in Latin, the works on mechanics of both Jordanus and Archimedes were printed and (in the case of Archimedes) retranslated, so that by about 1565 all the main ancient and medieval materials were available for a science of mechanics.

The non-mathematical translators and commentators, who were usually unfamiliar with Jordanus and Archimedes, praised the pseudo-Aristotelian *Mechanical Problems* for having established mechanics on a physical, i.e., a natural philosophical, principle involving the motion of heavy bodies. Most of the mathematicians concurred, but they went on to credit Archimedes with giving mechanics mathematical precision and certain demonstrations. Thus the Sicilian mathematician Francesco Maurolico, for instance, criticised those who had attempted to interpret the *Mechanical Problems* without the Archimedean principles of centres of gravity and equal moments, and asserted that Archimedes put Aristotle's conclusions into "the most ordered demonstration, in the way of the geometers" (*in demonstrationem ordinatissimam, Geometrarum more*).[10] Niccolò Tartaglia was one of the few who took a different tack. Having defined mechanics as concerned with augmenting powers for moving weights, he could find little use for Archimedes' purely statical approach using centres of gravity, despite his having been responsible for some of the first printings of translations of Archimedes' works. Nor was he satisfied with the pseudo-Aristotelian *Mechanical Problems*. Instead, he found the principles of mechanics in Jordanus and the medieval science of weights, in the powers and speeds of descending bodies.[11] Guidobaldo del Monte, while he had kind words to say of the *Mechanical Problems*, raised Archimedes to the pinnacle of mechanics. He followed Pappus of Alexandria explicitly "because he does not depart even a nail's breadth from the principles of Archimedes" (*quod ne latum quidem unguem ab Archimedeis principiis Pappus recedat*). Jordanus, in contrast, he accused of "disastrous errors"

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[9] See Paul Lawrence Rose and Stillman Drake, "The Pseudo-Aristotelian *Questions of Mechanics* in Renaissance Culture", *Studies in the Renaissance*, 18 (1971), 65-104; and W. R. Laird, "The Scope of Renaissance Mechanics", *Osiris* (2nd Series), 2 (1986), 43-68.

[10] Francesco Maurolico, *Problemata Mechanicarum appendice, et ad Magnetem, et ad Pixidem Nauticam pertinentia*, ed. Silvestro Maurolico (Messina, 1613), p. 10; quoted in Clagett, *Archimedes*, III, 785n.

[11] Niccolò Tartaglia, *Quesiti et inventioni diverse* (Venice, 1546; rpt. 1554; rpt. in facsimile Brescia: Ateneo, 1959), Books 7 and 8, tr. in Stillman Drake and I. E. Drabkin, *Mechanics in Sixteenth-Century Italy: Selections from Tartaglia, Benedetti, Guido Ubaldo, & Galileo* (Madison: Univ. Wisconsin Press, 1969), pp. 104-143.

(*quantas ruinas*).[12] Guidobaldo's *Mechanicorum liber* of 1577 was the most mathematically rigorous and comprehensive account of mechanics in its time, but this mathematical rigour and this strict adherance to Archimedes and Pappus had their price. His discussion of the balance, for instance, was excessively complicated by his insistance that the weights on the arms of a balance tend to converge towards the centre of the earth rather than act on parallel lines. As a result, he impuned the common rules based only on angles of descent adduced by Jordanus and Tartaglia:

> These men are, moreover, deceived when they undertake to investigate the balance in a purely mathematical way, its theory being actually mechanical; nor can they reason successfully without the true movement of the balance and without its weights, these being completely physical things, neglecting which they simply cannot arrive at the true cause of events that take place with regard to the balance.[13]

But the greatest shortcoming of Guidobaldo's mechanics, apparent only in retrospect, was his commonsensical insistance that the power to raise a weight must be greater than the power to sustain it. Accordingly, Guidobaldo's mechanics consisted of calculating from Archimedean principles the power to sustain any given weight, and then suggesting that some indefinite additional power must be added to move it. As a result, he notoriously preferred Pappus's erroneous statical analysis of equilibrium on inclined planes over the correct account by Jordanus, since Pappus had assumed that some finite power was needed to move a body on a horizontal surface, whereas Jordanus had appealed to speeds of motions.[14] Precisely what Guidobaldo left unspecified –the speeds of bodies when they are not in equilibrium– would later be the subject of Galileo's new science of motion.

The general attitude to the foundations of mechanics was expressed most clearly by Bernardino Baldi, another mathematician and sometime student of Guidobaldo's. According to Baldi, Aristotle had founded mechanics on the true physical principle of the balance –circular movement– but had left this principle indeterminate, so that Archimedes subsequently accepted this principle as a postulate and then went on to demonstrate mathematically that the lengths of the arms of the balance in equilibrium are inversely as the

[12] Guidobaldo dal Monte, *Mechanicorum liber*, Preface (Pesaro, 1577), tr. Filippo Pigafetta, *Le mechaniche* (Venice, 1581), tr. in Drake and Drabkin, *Mechanics in Sixteenth-Century Italy*, pp. 244, 246.

[13] "decipiuntur quinetiam, dum librae contemplationem mathematice simpliciter assummunt; cum eius consideratio sit prorsus mechanica: nec ullo modo absque vero motu, ac ponderibus (entibus omnino naturalibus) de ipsa sermo haberi possit: sine quibus eorum, quae librae accidunt, verae causae reperiri nullo modo possint", Guidobaldo, *Mechanicorum liber* (1577), ff. 17v-18r, tr. Pigafetta (1581), f. 16v, tr. Drake and Drabkin, *Mechanics in Sixteenth-Century Italy*, pp. 278-279.

[14] See Rose, *Italian Renaissance of Mathematics*, p. 233.

weights.[15] Thus the fundamental principle of mechanics was almost universally recognized to lie in the physical motions of heavy bodies as stated in the pseudo-Aristotelian *Mechanical Problems*. But at the same time Baldi, for one, pointed up the difficulty in applying a principle involving motion to conditions of equilibrium: "What speed", he asked, "can a stationary thing have?" (*Quae enim velocitas in re stante?*).[16] The Dutch mechanic Simon Stevin, who had hit on the brilliant intuitive proof for the law of equilibrium on inclined planes, flatly repudiated the application of Aristotle's principle to static equilibrium with the following syllogism:

> That which hangs still does not describe a circle;
> Two weights of equal apparent weight hang still;
> Therefore, two weights of equal apparent weight do not describe circles.[17]

The paradox of sixteenth-century Archimedean mechanics, then, was that it was apparently based on principles of motion, but its demonstrations concerned only equilibrium. How could it ever give rise to a science of motion?

Besides Tartaglia, as far as I know, only Giuseppe Moletti tried to keep motion in mechanics throughout, partly because he did not know the works of Archimedes. So instead of using Archimedean statics in his mechanics, he tried to demonstrate Aristotle's principle of circular movement with Euclidean geometry, and then he tried to apply this principle to motion in general, including the motion of projectiles and falling bodies.[18] In a general sort of way, this latter was exactly what Galileo would later do, though with considerably more success.

2 *De motu antiquiora,* 1590s

As is well known, Galileo was a great admirer of Archimedes. His first writings in mathematics were some hopeful theorems on centres of gravity in

[15] Bernardino Baldi, "Life of Archimedes," quoted in Drake and Drabkin, *Mechanics in Sixteenth-Century Italy*, pp. 14-15; see Rose, *Italian Renaissance of Mathematics*, p. 268.

[16] Bernardino Baldi, *In Mechanica Aristotelis problemata exercitationes* (Mainz: Vidua Ioannis Albini, 1612), p. 36; quoted and tr. in Rose, *Italian Renaissance of Mathematics*, p. 252 and 273n.

[17] "Ce qui demeure coy estant suspendu, ne descrit aucune circonference. Deux pesanteurs pendues en equilibre sont coyes. Deux pesanteurs pendues en equilibre donc, ne descrivent aucune circonference", Simon Stevin, *Les Œuvres Mathematiques de Simon Stevin*, tr. Albert Girard (Leiden, 1634; Vol. 4, rpt in facsimile Paris: Albert Blanchard, 1987), p. 501; *The Principal Works of Simon Stevin*, tr. C. Dikshoorn, 3 vols in 4 (Amsterdam, 1955), I, 509; the syllogism is translated in Drabkin and Drake, *Galileo Galilei on Motion and on Mechanics*, p. 143.

[18] See W. R. Laird, *The Unfinished Mechanics of Giuseppe Moletti* (Toronto: Univ. Toronto Press, 2000).

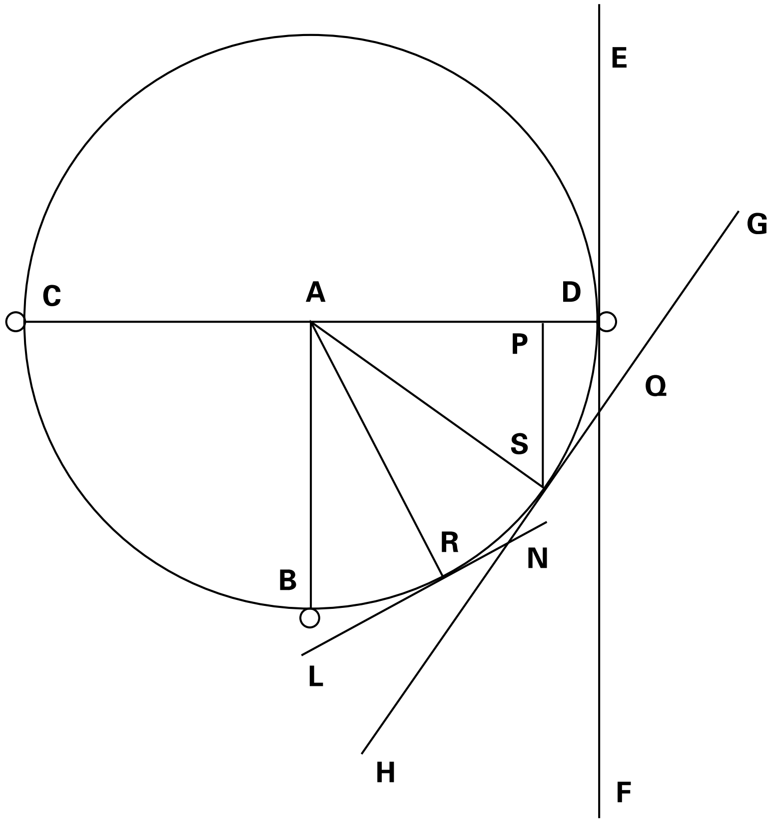

Fig. 2 Reduction of the inclined plane to the balance. Redraw from Galileo, De motu, Opere *1, 297.*

the manner of Archimedes; his first published work was on the hydrostatic balance and contained his reconstruction of Archimedes' solution to the Crown Problem; and his first attempt at a science of motion was to apply Archimedean hydrostatics to falling bodies, as if the signal characteristic of bodies in motion was that they move uniformly through a buoyant medium.[19] This last was in a group of works composed in the 1590s and known collectively as the "older works on motion" (*De motu antiquiora*), which Raymond Fredette discusses in the present volume. But Galileo was also heir to the Aristotelian mechanical tradition –he lectured on the *Mechanical Problems* at Padua in 1598– and in the longer prose version of *De motu* he adopted the central principle of circular movement in his treatments of the balance and of the speeds of bodies down inclined planes. The balance came up because the motions of the heavy and the light "cannot very well be further elucidated mathematically; they require rather a physical explanation" (*minus adhuc mathematice, et magis physice, declarari*

[19] On these early works, see Drake, *Galileo at Work*, pp. 6-14.

possunt) and this physical explanation was in fact the balance understood according to Aristotle's principle.[20] As for inclined planes, Galileo stated that the speeds of bodies down inclined planes "arise from known and obvious principles of nature" (*ex notis et manifestis naturae principiis ortum ducere*). In a brilliant insight, he grasped, contrary to Pappus and Guidobaldo, that ideally a body on a horizontal plane can be set in motion by any force however small, so that the power to sustain a weight and the power to move it are effectively equal. This enabled him immediately to reduce motion on inclined planes to the static equilibrium of the balance, in the following way (see Figure 2). He imagined that a weight on an inclined plane was in effect on the end of a balance, the plane being tangent to the circle made by the arm. He then showed that the speed and power of descent at any point of tangency (and thus along the tangent inclined plane) decrease as the ratio of the vertical drop to the length of the inclined plane.[21] In other words, he reduced the inclined plane to the bent lever and then to the equilibrium of the balance by comparing the speeds and powers of descent according to the principle of circular movement. Thus the principle of circular movement was for him one of the "known and obvious principles of nature".

3 L*e mecaniche,* 1600

The same principle and the same analysis of motion along inclined planes occurs in *Le mecaniche*, written about 1600 or 1601 for Galileo's private students at Padua. By this time, however, Galileo had refined his physical ideas sufficiently that he no longer spoke vaguely of speed and power, but rather of *momento*, by which he sometimes meant static moment and sometimes momentum.[22] At the beginning of *Le mecaniche* he presented a brilliant and original statical proof for the law of equilibrium along the lines of Archimedes', but then he went on immediately to confirm this law physically by appealing to the speeds of weights on arms of different lengths –in effect, the principle of circular movement. And again, when he came to consider the inclined plane (under the heading of the screw), he repeated the ingenious analysis that he had devised earlier in the *De motu antiquiora*, which I have just described.[23]

[20] Galileo, *De motu*, *Opere* 1, 257-260 (the phrase quoted is on p. 257), tr. in Drabkin and Drake, *Galileo on Motion and Mechanics*, pp. 20-23.

[21] Galileo, *De motu*, *Opere* 1, 296-302, tr. in Drabkin and Drake, *Galileo on Motion and Mechanics*, pp. 63-69.

[22] On Galileo's ideas of *momento*, see Paolo Galluzzi, *Momento: Studi galileiani* (Rome: Ateneo & Bizzarri, 1979).

[23] Galileo, *Le mecaniche*, *Opere* 2, 161-165, 179-183, tr. in Drabkin and Drake, *Galileo on Motion and Mechanics*, pp. 153-157, 170-175.

4 The New Science of Motion, 1638

Galileo's final appeal to the Aristotelian principle was in the culmination of lifetime's work devoted to motion. The new science of motion, of which Galileo was rightly so proud and on which his reputation largely stands today, was the extension of sixteenth-century Archimedean mechanics into motion. In effect, he picked up where Guidobaldo left off –with equilibrium– and established the rules governing the speeds of motions of bodies falling freely and along inclined planes. In the second book of *De motu locali*, printed in the Third Day of the *Discourses on Two New Sciences* (1638), Galileo established the science of accelerated motions on a single definition and a single postulate. The famous definition –that uniformly accelerated motions acquire equal speeds in equal times– has rightly been the focus of attention from historians.[24] From it followed immediately Galileo's most significant results in kinematics: the double-distance rule, the times-squared rule, and the odd-number rule. Galileo confirmed that the definition applies to actual falling bodies by appealing to measurements of motions on inclined planes.[25]

Compared to the definition, the postulate has received relatively less attention.[26] The postulate states that the final speeds of bodies descending from equal heights on planes however inclined are equal. In the original edition of *Two New Sciences*, Galileo confirmed it by appealing to an experiment with pendulums, in which the swinging bobs are seen to rise to their original heights even when the string is shortened by interposing an obstacle (see Figure 3). This, he argued, was because the *momenta* of speed they gain in falling are exactly sufficient to raise them back to the heights from which they fell, regardless of the arcs they follow. Although pendulum bobs follow circular arcs, Galileo asserted at the time that this principle was sufficiently evident to be applied to rectilinear inclined planes. He promised that its "absolute truth" (*la verità assoluta*) would be established later by conclusions based on it that exactly correspond to experience, though he never explicitly followed this up.[27]

But within a few months after *Two New Sciences* had appeared in print, several readers had already pointed out that the postulate had not been well established. Descartes, in a letter to Mersenne, listed a number of criticisms of the book, including the fact that the postuate was not proved and, as a result, that Galileo had "built everything on air" (*qu'il a tout basti en*

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[24] Galileo, *Discorsi e dimostrazioni matematiche intorno a due nuove scienze*, *Opere* 8, 198-205, tr. Stillman Drake, *Two New Sciences, Including Centers of Gravity and Force of Percussion* (Madision: Univ. Wisconsin, 1974; 2nd ed. Toronto: Wall & Thompson, 1989), 154-162; see, among many others, Winifred L. Wisan, "The New Science of Motion: A Study of Galileo's *De motu locali*", *Archive for History of Exact Sciences*, 13 (1974), 103-306.

[25] Galileo, *Discorsi*, *Opere* 8, 212-213, tr. Drake, *Two New Sciences*, pp. 169-170.

[26] See Wisan, "The New Science of Motion," esp. pp. 121-123.

[27] Galileo, *Discorsi*, *Opere* 8, 205-208, tr. Drake, *Two New Sciences*, pp. 162-164.

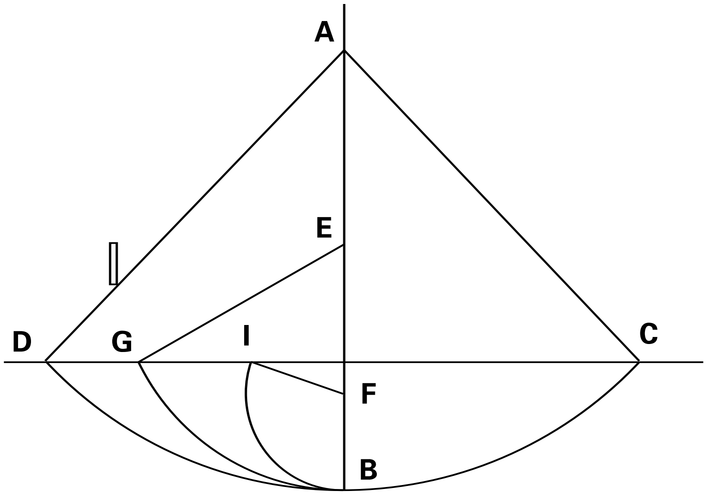

Fig. 3 Proof of the postulate by pendulums. Redrawn from Galileo, Discorsi, Opere *8, 206*

l'air).[28] Famiano Michelini in Siena wrote to express the difficulty Leopold de' Medici had in admitting the postulate as known.[29] And in a letter to Baliani, the now-blind Galileo himself conceded the point:

> That the principle I assume... does not, as you note, appear with that evidence that is required of principles to be postulated as known, I concede to you now... Know, then, that after my having lost my sight, and consequently my faculty of going more deeply into propositions and demonstrations more profound than those last discovered and written by me, I spent the nocturnal hours ruminating on the first and simplest propositions, reordering these and arranging them in better form and evidence. Among these it occurred to me to demonstrate the said postulate in the manner you will in time see, if I shall have sufficient strength to improve and amplify what was written and published by me up to now about motion by adding some little speculations...[30]

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[28] Descartes to Mersenne, 11 October 1638, in *Opere di Galileo Galilei* 17, # 3797, p. 390, tr. in Drake, *Galileo at Work*, pp. 387-392, on p. 391.

[29] Famiano Michelini to Galileo, 6 November 1638, in *Opere* 17, # 3809, pp. 399-400; see Drake, *Galileo at Work*, p. 395; see also *Opere* 17, #3816 and #3842.

[30] "che poi il principio che io suppongo, come V. S. nota, a faccie 166 [i.e., of the 1638 edition of the *Discorsi*], non gli paia di quella evidenza che si ricercherebbe ne' principii da supporsi come noti, gli lo voglio concedere per hora, ancorchè ella medesima faccia l' istessa suppositione, cioè che i gradi di velocità acquistati sopra l' orizonte da' mobili descendenti per diversi piani dalla medesima altezza siano equali. Hor sappia V. S. Ill.^{ma}, che doppo haver perso la vista, e per conseguenza la facoltà di potere andare internando in più profonde propositioni e dimostrationi che non sono le ultime da me trovate e scritte, mi sono andato nelle tenebre notturne occupando intorno alle prime e più semplici propositioni, riordinandole e disponendole in miglior forma et evidenza; tra le quali mi è occorso di dimostrare il sopra-

Later, in a letter to Benedetto Castelli, Galileo gave to his pupil Viviani, who had recently joined him in Arcetri, the credit for having pointed out the omission and prompting him to fill it. There Galileo called his new proof the "conclusive demonstration" (*la dimostratione concludente*) of the postulate, which was to be inserted into the *Discourses* "as a theorem essential to the establishment of the science of motion advanced by me" (*come teorema essentialissimo allo stabilimento delle scienze del moto da me promosse*).[31] The proof was apparently dictated to Viviani (a prose copy in his hand exists), who then put it into dialogue form and, much later, had it printed in the 1655-56 edition of Galileo's works under the title of a "posthumous addition of the author" (*Aggiunta postuma dell' autore*).[32] In this added proof, Galileo appealed to a result "demonstrated at length and conclusively" (*diffusamente e concludentemente dimostrato*) in "an old treatise on mechanics written at Padua for the use of his students" (*in un antico trattato di mecaniche, scritto già in Padova dal nostro Academico sol per uso de' suoi descepoli*) –i.e., *Le mecaniche*– that the impetuses of motions along inclined planes are inversely as the length of the planes, or in general,

> that when equilibrium (that is, rest) is to prevail between two moveables, their speeds or their propensions to motion –that is, the spaces they would pass in the same time– must be inverse to their weights, exactly as is demonstrated in all cases of mechanical movements.[33]

Then, by invoking the times-squared rule –that spaces traversed are as the squares of the times– which he had proved from the definition of accelerated motion alone, he proved the postulate –that the speeds acquired down inclined planes of equal heights are equal.[34] Thus the proof of the postulate is explicitly based on a conclusion from *Le mecaniche*, a conclusion that he had first proved almost 50 years earlier in the *De motu antiquiora* using the pseudo-Aristotelian principle of circular movement.

The only postulate, then, of Galileo's new science of motion was proved in mechanics, and its proof was based on the powers and speeds of motions

detto principio nel modo che a suo tempo ella vedrà, se mi succederà di havere tanto di forze che io possa migliorare et ampliare lo scritto e publicato da me sin qui intorno al moto..." Galileo to Gio. Battista Baliani, 1 August 1639, in *Opere* 18, # 3897, p. 78, tr. in Drake, *Galileo at Work*, pp. 400-401 (slightly altered).

31 Galileo to Bendetto Castelli, 3 December 1639, *Opere* 18, #3945, pp. 125-126, tr. in Drake, *Galileo at Work*, p. 405.

32 Favaro, *Opere* 8, 23-24; Favaro prints Viviani's prose copy in *Opere* 8, 442-445.

33 Galileo, *Discorsi*, *Opere* 8, 216, tr. Drake, p. 172; and "che quando debba seguire l' equilibrio, cioè la quiete tra essi mobili, i momenti, le velocità, o le lor propensioni al moto, cioè gli spazii che da loro passerebbero nel medesimo tempo, devon rispondere reciprocamente alle loro gravità, secondo quello che in tutti i casi de' movimenti mecanici si dimostra", *Opere* 8, 217, tr. Drake, p. 173.

34 Galileo, *Discorsi*, *Opere* 8, 218-219, tr. Drake, pp. 174-175.

descending along inclined planes, which in turn was reduced to the pseudo-Aristotelian principle of circular movement. While the definition of uniformly accelerated motion for falling bodies was confirmed only in the consequences it entailed for bodies on inclined planes, consequences that could then be observed by experiment, the postulate was demonstrated with a principle found in mechanics, one of the "known and obvious principles of nature". The conclusions of Galileo's new science of motion may have been purely kinematic and modelled after the statics of Archimedes, but they are ultimately founded upon the causal principle of circular movement –the principle of the balance– that Galileo and his predecessors in renaissance mechanics had drawn from the pseudo-Aristotelian *Mechanical Problems*.